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| Intersections between polygons |
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|  | INSTRUCTIONSIn order for the algorithm to work properly, the polygon should be convex.Polygons should be stored in a doubly connected edge list. | |  |
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|  | POLYGONS REPRESENTATION Polygon should be represented in 2 arrays, one for the upper-most chain(Blue), and the other for the down-most chain(Red).    Figure 1: Polygon 1  The left-most point is the start of the 2 chains, and the right-most point is the end of the 2 chains.  Since data is stored in a double linked list, is it accessible to get the chains easily. | Considering the blue chain’s array, each line should be stored in an independent index, such that the index should contain [StartPoint\_x, StartPoint\_Y, EndPoint\_x, EndPoint\_Y]  [point1\_X, point1\_Y, point2\_X, point2\_Y]    Figure 2: Upper chain of polygon 1  {[P11\_X, P11\_Y, P12\_X, P12\_Y],  [P21\_X, P21\_Y, P22\_X, P22\_Y],  [Pi1\_X, Pi1\_Y, Pi2\_X, Pi2\_Y]}  P11\_X represents the x-axis value of the starting point of line one.  P11\_Y represents the y-axis value of the starting point of line one.  P12\_X represents the x-axis value of the ending point of line one.  P12\_Y represents the x-axis value of the ending point of line one.      Figure 3: Lower chain of polygon 1  {[P61\_X, P61\_Y, P62\_X, P62\_Y],  [P71\_X, P71\_Y, P72\_X, P72\_Y],  [P81\_X, P81\_Y, P82\_X, P82\_Y]} |  |
|  | This is the projection of the polygon on the x-axis.  Figure 4: Projection of polygon 1on x-axis | |  |

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|  | Do the same steps done on the first polygon for each polygon that is passed to the algorithm.    Figure 5: Polygon 2  Polygon 2 will be divided into 2 chains, each chain will contain lines that shall be stored in an array | Figure 6: Upper chain of polygon 2  {[P11\_X, P11\_Y, P12\_X, P12\_Y],  [P21\_X, P21\_Y, P22\_X, P22\_Y],  [P31\_X, P31\_Y, P32\_X, P32\_Y]}      Figure 7: Lower chain of polygon 2  {[P41\_X, P41\_Y, P42\_X, P42\_Y],  [P51\_X, P51\_Y, P52\_X, P52\_Y]} |  |
|  | This is the projection of the polygon on the x-axis.    Figure 8: Projection of polygon 2 on x-axis  Next step is to combine all the projections together, polygon 1(Red)  chains will have same color, as well as polygon 2(Green).  This will be the final x-axis projection of both polygons.    Figure 9: Projection of both polygons on x-axis | |  |

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|  | SOLVING ALGORITHM Using Sweep Line Algorithm.  Create a sweep line that starts at the origin of the x-axis and moves to the next starting or ending point, check the intersection between lines that passes through the sweep line. If they intersect and they belong to different polygons, then there exists an intersection. | Figure 10: 2 polygons |  |
|  | EXAMPLE Consider the 2 convex polygons, both polygons are stored in double connected edge list.   1. Get the projection of all lines on the x-axis.      1. Create a sweep line that will pass through the x-axis.      1. Move the sweep line to the next end point until reach the end of last line.     Considering Figures 2 and 3, the sweep line is at the end of line 6. Now that line 6 ended, we use the turn left and turn right check. Line 6 is the main line here since it is the line that ended. | |  |

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|  | Consider the starting and ending point of line 1 to be (x1, y1) and (x2, y2), and consider the starting and ending point of line 6 to be (x3, y3) and (x4, y4).  Now we use the turn left and turn right to check if these 2 line segments intersect.  In the case of these 2 lines, there exist an intersection and the starting points of both lines, but since both lines belong to the same polygon, then the intersection is not counted. |
| Figure 11 |  |

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|  | Repeat step 3.    Considering Figures 2 and 3, the sweep line is at the end of line 1. So the main line now is line 1, the sweep line intersects lines 1 and 6. | |  |
|  | | Consider the starting and ending point of line 1 to be (x1, y1) and (x2, y2), and consider the starting and ending point of line 6 to be (x3, y3) and (x4, y4).  Now we use the turn left and turn right to check if these 2 line segments intersect.  In the case of these 2 lines, there exist NO INTERSECTION. | |
| Figure 12 | |  | |

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|  | Repeat step 3.    Considering Figures 2 and 3, the sweep line is at the end of line 2. So the main line now is line 2, the sweep line intersects lines 2 and 7. NO INTERSECTION | | |  | |
|  | Repeat step 3.    Considering Figures 2 and 3, the sweep line is at the end of line 7. So the main line now is line 7, the sweep line intersects lines 3 and 7. NO INTERSECTION | | |  | |
|  | Repeat step 3.    Considering Figures 6 and 7, the sweep line is at the start of lines 1’ and 4’. | | |  | |
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|  | Repeat step 3.    Considering Figures 2, 3, 6, and 7, the sweep line is at the end of line 3. So the main line now is line 3, the sweep line intersects lines 1’, 4’, 3, and 8. | | |  | |
|  | | Consider the starting and ending point of line 1’ to be (x1, y1) and (x2, y2),  Consider the starting and ending point of line 4’ to be (x3, y3) and (x4, y4).  Consider the starting and ending point of line 3 to be (x5, y5) and (x6, y6).  Consider the starting and ending point of line 8 to be (x7, y7) and (x8, y8).  Now we use the turn left and turn right to check if these line segments intersect with the main line.  In the case of these lines the main line does not intersect with any other line so NO INTERSECTION. | | | |
| Figure 13 | |  | | | |
|  | Repeat step 3.    Considering Figures 2, 3, 6, and 7, the sweep line is at the end of line 1’. So the main line now is line 1’, the sweep line intersects lines 1’, 4’, 4, and 8. | | |  | |
| Figure 14 | | Consider the starting and ending point of line 1’ to be (x1, y1) and (x2, y2),  Consider the starting and ending point of line 4’ to be (x3, y3) and (x4, y4).  Consider the starting and ending point of line 4 to be (x5, y5) and (x6, y6).  Consider the starting and ending point of line 8 to be (x7, y7) and (x8, y8).  Now we use the turn left and turn right to check if these line segments intersect with the main line.  In the case of these lines, there exist an intersection between the main line and line 4, since they do not belong to the same polygon, then there exist an INTERSECTION.  The man line and line 8 no intersection.  The main line and line 4’ exist a intersection, but since they belong to the same polygon, then no intersection.  INTERSECTION BETWEEN LINE 1’ AND 4. | | | |
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|  | Repeat step 3.    Considering Figures 2, 3, 6, and 7, the sweep line is at the end of line 4. So the main line now is line 4, the sweep line intersects lines 2’, 4’, 4, and 8. | |  | |
| Figure 15 | | Consider the starting and ending point of line 2’ to be (x1, y1) and (x2, y2),  Consider the starting and ending point of line 4’ to be (x3, y3) and (x4, y4).  Consider the starting and ending point of line 4 to be (x5, y5) and (x6, y6).  Consider the starting and ending point of line 8 to be (x7, y7) and (x8, y8).  Now we use the turn left and turn right to check if these line segments intersect with the main line.  In the case of these lines, there exist no intersection between the main line and any other line, so NO INTERSECTION.  . | | | |
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|  | Repeat step 3.    Considering Figures 2, 3, 6, and 7, the sweep line is at the end of lines 5 and 8. So the main lines now are lines 5 and 8, the sweep line intersects lines 2’, 4’, 5, and 8. | |  | |
| Figure 16 | | Consider the starting and ending point of line 2’ to be (x1, y1) and (x2, y2),  Consider the starting and ending point of line 4’ to be (x3, y3) and (x4, y4).  Consider the starting and ending point of line 5 to be (x5, y5) and (x6, y6).  Consider the starting and ending point of line 8 to be (x7, y7) and (x8, y8).  Now we use the turn left and turn right to check if these line segments intersect with the main line.  In the case of these lines, there exist no intersection between the first main line; line 5, and any other line.  There exists an intersection between the second main line; line 8, and line 4’, since they do not belong to the same polygon then there exist an INTERSECTION.  INERSECTION BETWEEN LINE 8 AND 4’. | | | |
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|  | Repeat step 3.    Considering Figure 7, the sweep line is at the end of line 4’. | | |  | |
|  | Repeat step 3.    Considering Figure 6, the sweep line is at the end of line 2’. | | |  | |
|  | Repeat step 3.    Considering Figures 6 and 7, the sweep line is at the end of lines 3’ and 5’. | | |  | |

### RESULT

Now that the sweep line is done, the algorithm detected that lines that intersect are (1’, 4) and (4’, 8).

### COMPLEXITY

The algorithm works in O(∑ni), when ni is the number of vertexes on polygoni .

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